A/2051 **Press Copy** 4305/MH

Total No. of Sheets used.........**02**.......... Total No. of Questions......**9**....................

Subject.................**Statistics**...............................Paper..............**II**...................

Title of the Paper..............**Probability Theory-II**...

Time allowed......**3 Hrs**.......Maximum Marks......**36**......Minimum Pass Marks..................

Instructions for the Candidates

Candidates are required to attempt five questions in all, selecting two questions from each sections A and B and the compulsory question of section C. All questions of sections A and B will carry 5 marks each where as section C will carry 16 marks.

Use of scientific non-programmable calculator is allowed.

Section-A

1. (a) Show that Binomial distribution is a limiting form of Hyper-Geometric distribution under certain conditions.

   (b) A deck of cards contains 20 cards: 6 red cards and 14 black cards. 5 cards are drawn randomly without replacement. What is the probability that exactly 4 red cards are drawn?

2. (a) Show that geometric distribution lacks memory.

   (b) Define Negative Binomial distribution. Find moment generating function. Hence find mean and variance of the distribution

3. Prove that Normal distribution is a limiting form of a Binomial distribution.

4. (a) Derive the p.d.f. of Chi-square distribution.

   (b) If X has a uniform distribution in $[0,1]$, find the distribution of $-2\log X$. Identify the distribution also.

Section-B

5. (a) Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ between them, then the correlation coefficient between X_1^2 and X_2^2 is given by ρ^2.

   (b) Let X and Y have bivariate normal distribution with parameters:

   $\mu_x = 4,\ \mu_y = 8,\ \sigma_x^2 = 1,\ \sigma_y^2 = 25$ and $Corr(X,Y) = \rho$. Find $P(X+Y) \le 15$ if $\rho = 0$.

6. Define bivariate normal distribution. If X and Y follow bivariate normal distribution, find the conditional distribution of Y given X.

Contd --- 2

7. (a) State and prove Chebychev's inequality.

(b) Use Chebychev's inequality to determine how many times an unbiased dice must be thrown in order that the probability will be at-least 0.80 that the ratio of the observed number of even to the number of odds will be between 0.4 and 0.6.

8. (a) Let $X_1, X_2, \ldots, X_n, \ldots$ be independent Bernoulli variates such that: $P(X_k = 1) = p_k, P(X_k = 0) = q_k = (1 - p_k), k = 1, 2, \ldots, n, \ldots$

Examine whether sequence $\left\{ \sum_{i=1}^{n} X_i \right\}$ follows central limit theorem.

(b) If $X_1, X_2, \ldots$ be i.i.d. Poisson variates with parameters λ. Use CLT to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + \ldots + X_n$; $\lambda = 2$ and $n = 75$.

Section-C

9. (i) If $X \sim B(n, p)$, show that $Cov.\left(\frac{X}{n}, \frac{n-X}{n} \right) = -\frac{pq}{n}$.

(ii) Find moment generating function of Geometric distribution.

(iii) State and prove additive property of Chi-square variates.

(iv) Define Beta distribution of second kind. Find its harmonic mean.

(v) State the important properties of Normal distribution.

(vi) Let $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Prove that $aX + bY, a \ne 0, b \ne 0$ is also a normal variate.

(vii) How large a sample must be taken in order that the probability will be at least 0.95 that $\bar{X}_n$ will be within 0.5 of μ (μ is unknown and $\sigma = 1$).

(viii) State weak law of large numbers.

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