A/2051

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5067/MH

COURSE: B. A./B.Sc. –Ist Year

Semester: II

Paper V: Partial Differential Equations

Time: 3 Hours Maximum Marks: 40

Instructions to Candidates:

Candidates are required to attempt five questions in all selecting two questions from each ***Section A*** *and* ***Section B*** *and compulsory question of* ***Section C****.*

Section-A

1. **(a)** Obtain a partial differential equation by eliminating the arbitrary function form the equation: $f\left(x^2+y^2, z-xy\right)=0.$ 3

   (b) Solve: $\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy.$ 3

2. **(a)** Find the general solution of the following partial differential equation:
$$\left(\frac{1}{z}-\frac{1}{y}\right)p+\left(\frac{1}{x}-\frac{1}{z}\right)q=\frac{1}{y}-\frac{1}{x}.$$ 3

   (b) Find the complete integral of $z^2\left(p^2+q^2\right)=x^2+y^2.$ 3

3. Find the complete integral of $p^2x+q^2y=z$ by Charpit's method. 6

4. Classify and reduce the equation $\dfrac{\partial^2 z}{\partial x^2}=x^2\dfrac{\partial^2 z}{\partial y^2}$ to canonical form. 6

Section-B

5. **(a)** Solve: $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial x\partial y}-2\dfrac{\partial^2 z}{\partial y^2}=(y-1)e^x.$ 3

   (b) Solve: $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial x\partial y}+\dfrac{\partial z}{\partial y}-z=\cos(x+2y).$ 3

6. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y=y_0\sin^3\left(\dfrac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement *y (x, t)*. 6

7. A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is $u(x,0)=\begin{cases} x & ;\ 0\le x\le 50 \\ 100-x & ;\ 50\le x\le 100 \end{cases}$. Find the temperature *u (x, t)* at any time. 6

8. Solve $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ which satisfies the conditions:
$$u(0,y)=u(l,y)=u(x,0)=0 \text{ and } u(x,a)=\sin\frac{n\pi x}{l}.$$ 6

Section-C (2 X 8 = 16)

9. **(a)** Find the partial differential equation of all spheres whose centre lies on z-axis.

(b) Find the general solution of $p + 3q = z + \cot(y - 3x)$.

(c) Find the equation of the surface which cuts orthogonally the system of surfaces $k(z+2) - 2xz - 3yz = 0$, where k is an arbitrary constant, and passes through the circle $z = 0, x^2 + y^2 = 9$.

(d) Show that the equation $z_{xx} + 2xz_{xy} + (1 - y^2)z_{yy} = 0$ is elliptic for all values of x, y in the region $x^2 + y^2 < 1$, parabolic on the boundary and hyperbolic outside the region.

(e) Find the general solution of the partial differential equation:

$$\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial^2 y} + 6\frac{\partial^3 z}{\partial y^3} = 0$$

(f) Solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y} + x^3$.

(g) Solve: $t - xq = x^2$.

(h) Solve: $(DD' - D - D' - 1)z = xy$.

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