AS-2051

CALCULUS-II PAPER -IV

SEMESTER -II

TIME :3 HOURS   M:M: 40

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

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SECTION-A

I(a) Show that $\int_0^1 dx \int_0^1 \frac{x^2-y^2}{x^2+y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2-y^2}{x^2+y^2} dx$. (3)

I(b) Write an equivalent double integral with order of integration

reversed for $\int_0^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y\, dx\, dy$. Hence evaluate the double integral. (3)

II(a) By changing into polar co-ordinates, evaluate $\iint \sqrt{a^2-x^2-y^2}\,dxdy$

$over\ the\ circle\ x^2+x^2 \le ax\ in\ the\ positive\ quardrant\ where\ a > 0.$ (3)

II(b) Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay, a > 0.$ (3)

III(a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 \frac{dz\, dy\, dx}{\sqrt{x^2+y^2}}$. (3)

III(b) Show that the entire volume of solid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} + \left(\frac{z}{c}\right)^{\frac{2}{3}} = 1\ is\ \frac{4\pi abc}{35}$. (3)

IV Find the centre of gravity of a right circular cone of base radius r and

height h with uniform density unity. (6)

SECTION-B

V(a) Let $\vec{a}$ and $\vec{b}$ be two nom-zero vectors, then prove that $\vec{a}$ and $\vec{b}$ are perpendicular

if and only if $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ (3)

V(b) If $\vec{a}, \vec{b}\ and\ \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$, show that

the three vectors $\vec{a}$, $\vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}| = |\vec{c}|, |\vec{b}| = 1$ (3)

VI(a) Find equation of plane passing through line of intersection of the planes

$\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k}) = 6$ and $\vec{r}.(2\hat{\imath}+3\hat{\jmath}+4\hat{k}) = -5$ and the point $(1,1,1)$. (3)

VI(b)Find the equation of the line(vector and cartesian both) which is parallel

To the vector $2\hat{\imath}-\hat{\jmath}+3\hat{k}$ and which passes through the point $(5,-2,4)$ (3)

Contd— 2

VII State and prove Green's Theorem in a plane. (6)

VIII(a) Verify Stoke's Theorem for $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (3)

VIII(b) Apply Divergence theorem, to evaluate $\iint \vec{A}.\vec{n}\, dS$ *over the surface of region bounded by* $x^2 + y^2 = 4, z = 0$ *and* $z = 3$ (3)

SECTION-C

IX(a) Evaluate $\iint \sin \pi(x^2 + y^2)dx\, dy$ over the circle $x^2 + y^2 \leq 1$.

IX(b) Define Moments of Inertia of solid of mass M continuously distributed with mass density $\mu(x, y, z)$ *throughout a region* $V \subset \mathbf{R}^3$

IX(c) Evaluate $\iiint (z^5 + z)\, dxdydz$ *over region*

$$V = \{(x, y, z): x^2 + y^2 + z^2 \leq 1\}$$

IX(d)Find the moment of inertia of a square region of unit density about one of its sides, the side being 2a.

IX(e)Find the circulation of $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ round the curve C, where C is the circle $x^2 + y^2 = 1, z = 0$ in xy-plane.

IX(f) Find Find the vector equation of the line passing through the points $A(-1, 0, 2)$ *and* $B(3, 4, 6)$

IX(g) If $\vec{a}$ and $\vec{b}$ are unit vectors, and θ is angle between them, find $\frac{1}{2}|\vec{a} - \vec{b}|$ in terms of θ.

IX(h) Define the flux of a vector point function across a surface.

(2× 8 = 16)

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