AS-2051
PARTIAL DIFFERENTIAL EQUATION -V
SEMESTER -II

TIME :3 HOURS **M:M: 40**

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

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SECTION-A

I(a). Form a partial differential equation by elimination of arbitrary constants from following relation: $(z + a^2)^3 = (x + ay + b)^2$ (3)

I(b). Form a partial differential equation by elimination of arbitrary function from following relation: $f(lx + my + nz,\ x^2 + y^2 + z^2) = 0.$ (3)

II (a).Find the integral surface of the partial differential equation

$p + q = 1, which\ passes\ through\ x = 0, y^2 = z$ (3)

II (b). Find the general solution of following lagrange's linear equation

$(3z - 5y)p + (5x - 2z)q = 2y - 3x$ (3)

Iii(a) Find the surface passing through the parabolas, $y^2 = 4ax,\ z = 0\ and$ $y^2 = -4ax,\ z = 1$ and satisfying the partial differential equation

$xr + 2p = 0$ (3)

IIII(b).Find the equation of surface which cuts surfaces of the system

$z(x + y) = \lambda(3z + 1)$ orthogonally. (3)

IV Using Charpit's method to find complete solution of the following partial differential equation $z = p^2x + q^2y.$ (6)

SECTION-B

V (a) Find the general solution of the partial differential equation

$(D_x{}^3 - 7D_xD_y{}^2 - 6D_y{}^3)(z) = \cos(x + 2y) + e^{-2x+y}$ (3)

V (b) Find the general solution of the partial differential equation

$r + s - 6t = y\cos x$ (3)

VI (a) Find the general solution of the partial differential equation

$(D_x{}^4 + D_y{}^4)(z) = 0$ (3)

VI (b) Find the general solution of the partial differential equation

$r - q = e^{x+y}$ (3)

VII Obtain the solution of one dimensional heat equation

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, by the method of separation of variables. (6)

VIII Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, *subject to*

$u(0,y) = u(l,y) = o$ *where* $0 \leq y \leq m$ *and* $u(x,m) = 0$, $u(x,0) = F(x)$

where $0 \leq x \leq l$. (6)

SECTION-C

IX (a). Define Lagrange's Linear partial differential equation.

IX (b) Solve the partial differential equation for complete solution

$p^2 + q^2 = z$

IX (c) Define the singular solution of partial differential equation.

IX(d) State the conditions under which the partial differential equation

$$rR + sS + tT + f(x,y,z,p,q) = 0$$

to be classified as Parabolic, Elliptic or Hyperbolic, where R, S and T are continuous functions of x and y

IX (e) Write a short note on Method of separation of variables for solving second order partial differential equation of the form

$pP + qQ + rR + sS + tT = W$, where P,Q,R,S,T,W are functions of x, y only

IX (f) Find the general solution of the partial differential equation $r - 5q + 20z = 0$

IX (g) Find the general solution of the partial differential equation $(D_x{}^3 - D_y{}^3)z = xy^2$

IX(h)Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection

$f(x) = k(\sin x - \sin 2x)$

$2 \times 8 = 16$

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