AS-2051
ANALYTIC GEOMETRY -VI
SEMESTER -II

TIME :3 HOURS **M:M: 40**

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

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SECTION-A

I Find axis, latus rectum, vertex and focus of the parabola$x^2 + 2xy + y^2 - 2x - 1 = 0$. (6)

II Trace the conic $36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$. (6)

III(a) Prove that the equation of the normal to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point P

with vectorial angle α is $\frac{le\sin\alpha}{r(1+e\cos\alpha)} = \sin(\theta - \alpha) + e\sin\theta$. (3)

III(b) Prove that the angle θ between the lines represented by $ax^2 + 2hxy + by^2 = 0$

is given by $\tan\theta = \frac{2\sqrt{h^2-ab}\sin\omega}{a+b-2h\cos\omega}$, where ω is the angle between the coordinate axes. (3)

IV(a) Find the equation of the ellipse with axes as any pair of conjugate diameters. (3)

IV(b) If by any change of oblique axes, the expression $ax^2 + 2hxy + by^2$ $changes\ to$

$$a'x^2 + 2h'xy + b'y^2, then\ \frac{a + b - 2h\cos\omega}{(\sin\omega)^2} = \frac{a' + b' - 2h'\cos\omega'}{(\sin\omega\,')^2}\ and$$

$\frac{ab-h^2}{(\sin\omega)^2} = \frac{a'b'-h'^2}{(\sin\omega\prime)^2}$ $,where\ \omega\ and\ \omega'$ are the inclinations of the two sets of axes. (3)

SECTION-B

V(a) Find the equation of two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$, which

passes through the line $x + y = 6, x - 2z = 3$. (3)

V(b) P is a variable point on a given line and A, B, C are its projections on the axes.

Show that the sphere OABC passes through a fixed circle. (3)

VI(a) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at

the point $P(1, -2, 1)$ $and\ also\ cuts\ orthogonally\ the$

$sphere$ $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$ (3)

VI(b) A sphere of constant radius r passes through the origin O and cut the axes in A, B, C.

Prove that The centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4r^2$. (3)

VII(a) Find equation of right circular cone with vertex $(2, -3, 5)$, semi vertical angle $30°$

and whose axis makes make equal angle with coordinate axes. (3)

VII(b) Show that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ with vertex at $(2, 4, 1)$ in a rectangular hyperbola. (3)

VIII(a) Find equation of right circular cylinder of radius 2 whose axis passes through the point $(1, 2, 3)$ and has the direction ratios $\langle 2, -3, 6 \rangle$. (3)

VIII(b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 1 = 0$ whose generators are parallel to the line having direction ratios $\langle 1, 1, 1 \rangle$. (3)

SECTION-C

IX(a) Prove that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at a point P with Vectorial angle α is $\frac{l}{r} = \cos(\theta - \alpha) + e\cos\theta$

IX(b) Prove that lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$ make equal angle with x-axis if $m_1 + m_2 + 2m_1 m_2 \cos\omega = 0$.

IX(c) Define Conic Section.

IX(d) Prove that equation of a circle with centre (h, k) and radius a is

$$x^2 + y^2 + 2xy\cos\omega - 2(h + k\cos\omega)x - 2(k + h\cos\omega)y + h^2 + k^2 + 2hk\cos\omega - a^2 = 0$$

where ω is the angle between the axes

IX (e). Find the centre of section of the sphere $x^2 + y^2 + z^2 = 25$ by the plane $2x + y + 2z = 9$.

IX (f). Find the length of the tangent drawn from the point (1, -1, 3) to the sphere

$$3(x^2 + y^2 + z^2) + 3x - y + 2z + 2 = 0$$

IX (g). Define right circular cone.

IX(h). Find the equation of a right circular cylinder having axis as z-axis and radius r

(2× 8 = 16)

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