A-2051
ALGEBRA-I PAPER-IV)
SEM (II) PVT (MAY-18)

TIME: 3 HOURS **M:M:100**

5891/MH

Note: The candidates are required to attempt two questions each from sections A and B carrying 15 marks each and entire section C is consisting 8 short answer type questions carrying 5 marks each.

SECTION-A

1. **(a)** Express the matrix $A=\begin{bmatrix} 2-i & 3 & 1+i \\ -5 & 0 & -6i \\ 7 & i & -3+2i \end{bmatrix}$ as the sum of hermitian and skew hermitian matrices. (10)

(b) Prove that the rank of transpose of a matrix A is the same as that of the original matrix A. (5)

2. **(a)** Find the inverse of a matrix $A=\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 5 & 2 & 3 \end{bmatrix}$ using elementary transformations. (8)

(b) Prove that the characteristic roots of hermitian matrix are real (7)

3. (a) Reduce to row reduced echelon form the matrix A= $\begin{bmatrix} 0 & 1 & 3 & -1 & 4 \\ 2 & 0 & -4 & 1 & 2 \\ 1 & 4 & 2 & 0 & -1 \\ 3 & 4 & -2 & 1 & 1 \\ 6 & 9 & -1 & 1 & 6 \end{bmatrix}$ (8)

(b) Find the characteristic equation and eigen values of the matrix A= $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ (7)

4. Show that the only real value of 'a' for which the equation

$x+2y+z=ax$

$x+y+2z=ay$

$2x+y+5=az$

have a non trivial solution is 4. Also solve the equation for a=4 (15)

Contd...2

SECTION-B

5. **(a)** Solve the equation $2x^3 - 15x^2 + 37x - 30 = 0$, when its roots are in A.P. (8)

(b) Solve the equation $4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$, two of its root are equal but opposite in sign (7)

6. **(a)** Find the equation whose roots are squares of differences of the roots of the cubic equation $ax^3 + 3bx^2 + 3cx^2 + d = 0$ (8)

(b) Use descrates method to solve $x^3 - 18x - 35 = 0$. (7)

7. **(a)** Find the seventh roots of unity and show that sum of roots vanishes. (8)

(b) If $\sin(u + iv) = x + iy$ then prove $x^2 \operatorname{cosec}^2 u - y^2 \sec^2 u = 1$. (7)

8. **(a)**Find the sum of the series $\sin x + \frac{1}{2}\sin 2x + \frac{1}{2^2}\sin 3x + \ldots\ldots\ldots\ldots$ up to infinity. (7)

(b) Find the sum of cosines of n angles in A.P. (8)

SECTION-C

(8x5=40)

9.

(a) Examine the system of vectors (1, 3, 2), (1, -7,-8), (2, 1, -1) are Linearly Independent or dependent.

(b) Define symmetric and skew symmetric matrix with examples

(c) If P and Q is unitary matrix, prove that QP is also unitary matrix.

(d) Find the value of k so that the equations have unique solution $x + 2y - kz = -1$.

$$3x - y + kz = 1, \quad 2x + y + z = 2,$$

(e) Find the nature of the roots of the equation $x^5 - 3x^4 + 4x^3 + 2x^2 + x - 8 = 0$.

(f) Split up $e^{(6+5i)^2}$ into real and imaginary part.

(g) Prove that $\sin(\log i^i) = -1$.

(h) State Cayley's Hamilton theorem. Verify it with suitable example.

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