

PC-5900/MH

AS/2051

MATHEMATICS – XI(a)

(For Biology Students)

(Semester-II)

(May-2019)

Time : Three Hours]

[Maximum Marks : 56

Note : Attempt *two* questions each from Section A and B carrying 10 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION – A

I. (a) Prove that

$$\frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}} = 2 - \sqrt{3} .$$

(b) Prove that $\tan 65^\circ = 2 \tan 40^\circ + \tan 25^\circ$. (5+5=10)

II. (a) Prove that

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a .$$

(b) Prove that

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}. \quad (5+5=10)$$

III. (a) Differentiate $(x^x)^x + x^{(x^x)}$.

(b) If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ then show that

$$2x \frac{dy}{dx} + y = 2\sqrt{x}. \quad (5+5=10)$$

IV. (a) If

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots + \infty}}}$$

then prove that

$$(2y-1) \frac{dy}{dx} = \frac{1}{x}.$$

(b) If $\sqrt{x} + \sqrt{y} = a$ then prove that $\frac{dy}{dx} + \frac{dx}{dy} = 1$.

(5+5=10)

SECTION – B

V. (a) Evaluate

$$\int \frac{4x+1}{\sqrt{2x+1}} dx.$$

(b) Evaluate

$$\int (x+1)\sqrt{x+2} dx. \quad (5+5=10)$$

VI. If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, prove that $A^{-1} = A^2 - 6A + 11I$. (10)

VII. Using Cramer's Rule, solve the following system of equations :

$$x + 3y - 2z = 5$$

$$2x + y + 4z = 8$$

$$6x + y - 3z = 5. \quad (10)$$

VIII. Using matrix method, solve the following system of equations :

$$x + y + z = 6,$$

$$2x - y + z = 3,$$

$$x - 2y + 3z = 6. \quad (10)$$

SECTION - C

IX. (a) Define symmetric matrix with an example.

(b) Define row matrix and column matrix with an example.

(c) Compute AB if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 6 & 4 \\ 4 & 7 & 5 \end{bmatrix}$.

(d) Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.

(e) Evaluate $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$.

(f) Find $\frac{dy}{dx}$ when $y = x^3 + 7x^2 + 4x + 9$.

(g) Integrate $f(x) = x^4 + x^{-5}$.

(h) Find a matrix X such that $A + 2B + X = 0$ where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}. \quad (8 \times 2 = 16)$$
