PC-5920/MH

AS-2021

ALGEBRA–I
Paper-IV
(Semester–II)
(May-19)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION – A

I. (a) Define an orthogonal matrix.

Is $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ orthogonal?

(b) Prove that the chsracteristics roots of a Hermitian matrix are real.

II. (a) If $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ -3 & -6 & -4 \end{bmatrix}$, find the minimal polynomial of A. Find A^{-1} using the minimal polynomial.

(b) Use Gauss Jordon method to find the Inverse of

$$\begin{bmatrix} 1 & -2 & 3 \\ -2 & -1 & 0 \\ -4 & -2 & 5 \end{bmatrix}.$$

III. (a) State the conditions under which a system of non-homogeneous equations have :

(i) No solution

(ii) A unique solution

(iii) Inifinite number of solutions. (2)

(b) Find all the eigenvalues and eigen vectors of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

over R. Is this matrix diagonalisable? Justify. (3)

(c) Prove that the column rank of a matrix is the same as its rank. (1)

IV. (a) Define similar matrices and prove that the similar matrices have same characteristics polynomial and hence same eigen values. (2)

(b) State and prove Cayley-Hamilton theorem. Use Cayley-Hamilton theorem to Find A^2 when

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$ (4)

SECTION – B

V. (a) Prove that i^i is wholly real and find its principal value. Also show that the values of i^i form a G.P.

(b) Use Cardon's method to solve

$$x^3 - 6x^2 - 6x - 7 = 0 \cdot$$ (3,3)

VI. (a) Use Descarte's method to solve

$$x^4 - 3x^2 - 42x - 40 = 0 \cdot$$

(b) Prove that $\tan\left(i \log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2-b^2}$. (3,3)

VII. (a) Find A and solve the equation :

$$40x^4 + \lambda x^3 - 21x^2 - 2x + 1 = 0$$

given that roots are in H.P.

(b) Find the equation of the squared difference of the roots of the cubic $x^3 - 9x^2 + 23x - 15 = 0$. (3)

VIII. (a) Sum to n terms the series

$$1 + x\cos\alpha + x^2\cos 2\alpha + \ \cdots\cdots\cdots$$

Also find the sum to infinity when $|x| < 1$.

(b) If the product of two roots of

$$x^4 + px^3 + qx^2 + r\,x + s = 0$$

is equal to product of the other two, then show that

$r^2 + p^2 s$. (3,3)

SECTION – C

IX. (a) Examine the linear independence or dependence of

$(2, -1, 3), (8, 2, 0)$ and $(0, 1, -2)$.

(b) If P is unitary and Q is Hermitian, show that $P^{-1}QP$ is Hermitian.

(c) What is Gregory series?

(d) If $x = \cos\theta + i\sin\theta$ then $\dfrac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$.

(e) Show that the equation

$$2x^7 + 3x^4 - 3x + k = 0$$

has at least four imaginary roots for all values of k.

(f) Show that at least one eigen value of every singular matrix is zero.

(g) Find the principal value of $\log(-1+i) - \log(-1-i)$.

(h) Prove that a real matrix is unitary iff it is orthogonal.

(8×2=16)