PC-5923/MH

B-2051

ALGEBRA–I
Paper–IV
(Semester–II)
(May-18)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B. Section C will be compulsory.

SECTION – A

I. (a) Show that every square matrix is uniquely expressible as the sum of a symmetric and a skew symmetric matrix.

(b) Express $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (3+3)

II. (a) Prove that the characteristic books of Skew-Hermitian matrix are either zero or purely imaginary.

(b) Diagonalize the matrix,

$$A = \begin{bmatrix} 1 & 3 & 6 \\ -3 & -5 & -6 \\ 3 & 3 & 4 \end{bmatrix}$$ (2+4)

III. (a) Using matrix method, show that the equations,

$$5x + 3y + 7z = 4,$$

$$3x + 26y + 2z = 9,$$

$$7x + 2y + 10z = 5$$

are consistent and solve them.

(b) For $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, find the value of

$A^5 + 5A^4 - 6A^3 + 2A^2 - 4A + 7I$

by using Cayley Hamilton theorem. (3+3)

IV. Find Non-Singular matrices P and Q such that PAQ is the normal form for the matrix

$$\begin{bmatrix} 3 & -1 & 2 & 1 \\ 1 & 4 & 6 & 1 \\ 7 & -11 & -6 & 1 \\ 7 & 2 & 12 & 3 \end{bmatrix}.$$ (6)

SECTION – B

V. (a) Construct a cubic polynomial $f(x)$ having the properties :

(i) $f(x)$ is monic,

(ii) $f(x) = 0$,

(iii) $f(0) = -8$,

(iv) Sum of roots of $f(x) = 7$.

(b) Solve $18x^3 + 81x^2 + 21x + 60 = 0$, one root being half the sum of other two. (3+3)

VI. (a) If α, β, γ are the roots of $x^3 + 3x + 2 = 0$, form an equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2$ and $(\alpha - \beta)^2$.

(b) Hence, show that the equation has a pair of imaginary roots. (3+3)

VII. (a) Solve $x^3 + 3x - 14 = 0$ by Cardan's method.

(b) Prove that $(a+ib)^{m/n} + (a-ib)^{m/n} = 2(a^2+b^2)^{m/2n}$.

$\cos\left[\frac{m}{n} \bullet \tan^{-1}\left(\frac{b}{a}\right)\right]$. (3+3)

VIII. Solve $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method. (6)

SECTION – C

(Compulsory Question)

IX. (a) Show that every invertible matrix has a unique inverse.

(b) Prove that rank of matrix and its transpose is equal.

(c) Find the quotient and remainder when

$6x^4 + 11x^3 + 13x^2 - 3x + 27$ is divided by $3x + 4$.

(d) Find the equation whose roots are $\alpha_1, \alpha_2, \alpha_3$ of equation with sign changed.

(e) Find the nature of the roots of the equation

$x^5 - 7x^4 + 6x^3 + 5x^2 + 4x - 8 = 0$.

(f) Find the real and imaginary parts of tan h $(x + iy)$

(g) Prove that $i^i = e^{-(4n+1)\pi/2}$.

(h) State De-MoIver's Theorem. (8×2=16)