PC-5935/MH

A/2051

ALGEBRA–I

Paper–IV

(Semester–II)

(Syllabus May-2020)

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each.

SECTION – A

I. (a) Prove that every square matrix can be expressed in one end only one way as $P + iQ$, where P and Q are Hermitian.

(b) Express the matrix $\begin{bmatrix} 2+3i & 1+i \\ 3-4i & 0 \end{bmatrix}$ as $P + iQ$, where P, Q are Hermitian matrices. (3+3=6)

II. (a) Write the system of equations in matrix form;

$$x+y+z=7$$
$$x+2y+3z=16$$
$$x+3y+4z=22$$

and find inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

(b) Hence, solve the given system of equations.

(3+3=6)

III. (a) Find the characteristic equation and eigen values of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. What is the spectrum of the matrix?

(b) Verify that the matrix in above question part (a), satisfies its characteristic equation. (3+3=6)

IV. Investigate for what values of λ and μ, the equation,

$$x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu;$$

have (i) no solution (ii) unique solution (iii) infinite number of solution. (6)

SECTION – B

V. (a) Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, the roots being A.P.

(b) Solve the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$, given that one root is $2 + \sqrt{3}$. (3+3=6)

VI. (a) Solve the equation $x^4 - 7x^3 + 18x^2 - 2x + 12 = 0$ if product of two of its root is 6.

(b) If α, β, γ are the roots of the cubic $x^3 + 3x + 2 = 0$, find an equation whose roots are $(\alpha - \beta)(\alpha - \gamma)$, $(\beta - \gamma)(\beta - \alpha)$, $(\gamma - \alpha)(\gamma - \beta)$. (3+3=6)

VII. (a) Solve

$$2x^3 + 3x^2 + 3x + 1 = 0$$

by Cardon's method.

(b) Solve

$$x^4 + 8x^3 + 9x^2 - 8x - 10 = 0$$

by Discarte's method. (3+3=6)

VIII. (a) Using De-Moivre's theorem solve the equation

$$x^4 + x^3 + x^2 + x + 1 = 0 \cdot$$

(b) Expand $\cos^8 \theta$ in a series of cosines of multiplas of θ. (3+3=6)

SECTION – C

IX. (a) If P and Q are symmetric matrices, prove that $PQ - QP$ is skew symmetric matrix.

(b) Find rank of $\begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \\ 3 & 5 & 4 \end{bmatrix}$.

(c) Show that $\{(1, 2, 3), (2, -2, 0)\}$ form a linearly independent set.

(d) Prove that characteristic roots of unitary matrix are of unit modulus.

(e) Construct a polynomial equation over rationals of degree 4 whose roots are $\sqrt{3}$ and $1 + 2i$.

(f) Plot the cube roots of unity in an Argand diagram and find their product.

(g) Determine the general value of $\log(-i)$.

(h) Prove that $\sinh^{-1} x = \cosh^{-1}(\sqrt{1+x^2})$. (8×2=16)