AS-2051
ANALYTIC GEOMETRY -VI
SEMESTER -II

TIME :3 HOURS **M:M: 40**

NOTE : The candidates are required to attempt two questions each from Section A and B Section C will be compulsory .

708/MH

SECTION-A

I Trace the parabola $x^2 - 4xy + 4y^2 + 10x - 8y + 13 = 0$. Also find the vertex, directrix and the focus. (6)

II Define the director circle of a conic and find the equation of the director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$. (6)

III(a) Prove that the centre of the conic section $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$. (3)

III(b) Find center and radius of circle $x^2 + 2xy\cos\omega + y^2 - 2gx - 2fy = 0$, axis being inclined at $an\ angle\ \omega$. (3)

IV(a) The length of perpendicular drawn from point (x_1, y_1) to the line $ax + by + c = 0$ referred to the axis inclined at an angle ω is $\frac{|(a_1x+b_1y+c_1)\sin\omega|}{\sqrt{a^2+b^2-2ab\cos\omega}}$. (3)

IV(b) Prove that the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$. (3)

is $\tan^{-1}\left(\frac{(m_1-m_2)\sin\omega}{1+(m_1+m_2)\cos\omega+m_1m_2}\right)$, where ω is the angle between the axes. (3)

SECTION-B

V. A sphere of constant radius r passes through the origin O and cut the axes in A, B, C. Prove that the locus of the foot of perpendicular line from O to the plane ABC is given by $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4r^2$ (6)

VI (a). Tangent plane at any point of sphere $x^2 + y^2 + z^2 = r^2$ meets the coordinate axes in A, B and C. show that locus of point of intersection of planes drawn parallel to coordinate planes is the surface $x^{-2} + y^{-2} + z^{-2} = r^{-2}$. (3)

VI(b). Find equation of sphere which touches the plane $3x + 2y - z + 2 = 0$ at point $P(1, -2, 1)$ and also cut orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. (3)

VII(a). If θ is the semi-vertical angle of a right circular cone which passes through the lines OX, OY and $x = y = z, show\ that$ $\cos\theta = (9 - 4\sqrt{3})^{-\left(\frac{1}{2}\right)}$ (3)

Contd — 2

VII(b). Prove that the plane $x + y + z = 0$ *cuts the cone* $ayz + bzx + cxy = 0$

in perpendicular lines if $a + b + c = 0$ (3)

VIII(a). Find equation of right circular cylinder whose guiding curve is the circle

passing through the points $(2, 0, 0), (0, 2, 0), (0, 0, 2)$. (3)

VIII(b). Prove that enveloping cylinder of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ with generators

parallel to line $\frac{x}{0} = \frac{y}{\pm\sqrt{a^2-b^2}} = \frac{z}{c}$ meet the XY plane in a circle. (3)

SECTION-C

IX(a) Prove that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ will touch the conic $\frac{l}{r} = 1 + e\cos\theta$

If $(A - e)^2 + B^2 = 1$.

IX(b) If the tangent and normal at a point P of a conic meet the axis in T and G, then prove

that $\frac{1}{SG} - \frac{1}{ST}$ is constant, where S is the focus of the conic.

IX(c)I f the axes are inclined at an angle ω, *then prove that distance between the points*

(x_1, y_1) *and* (x_2, y_2) *is* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(y_2 - y_1)\cos\omega}$

IX(d) Prove that lines $y = m_1x + c_1$ and $y = m_2x + c_2$ make equal angle with x-axis

If $m_1 + m_2 + 2m_1m_2\cos\omega = 0$.

IX (e). Prove that the plane $x + 2y - z = 4$ *cuts the sphere* $x^2 + y^2 + z^2 - x + z - 2 = 0$

In a circle of radius unity.

IX (f). If a tangent plane to the sphere $x^2 + y^2 + z^2 = p^2$ makes intercepts a, b, c on

the coordinate axes, prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

IX (g) Prove that semi vertical angle of a right circular cone having three mutually perpendicular generators is $\tan^{-1}\sqrt{2}$.

IX(h). Define right circular cylinder.

7-8/MH (2× 8 = 16)